On the optimality of the optimal policies for the deterministic EPQ with partial backordering

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Abstract

In a note published in Omega [Zhang RQ. A note on the deterministic EPQ with partial backordering. Omega 2009;37(5):1036-8], the amended decision procedure for the Pentico et al.'s EPQ with partial backordering (EPQ-PBO) is proposed, by developing another critical value of the backordering rate. However, there is a case when a decision made with this amended procedure is not optimal, which will be shown in this paper. A new decision procedure will be proposed based on the derived necessary and sufficient conditions for considering the policy of losing all sales or the policy to meet all demand as possible optimal decisions. The proposed decision procedure is adapted for one of the extensions of the EPQ-PBO.

Key words. EPQ, partial backordering, decision procedure, optimal policy

AMS subject classification. 90B30, 90B50

1 Introduction

One of the widely used models for inventory control, the classic square root economic order quantity (EOQ) model, during the past years has been the basis for many other models. When the assumption of instantaneous replenishment is replaced with the assumption that the replenishment order is received at a constant finite rate over time, EOQ is extended to the economic production quantity (EPQ) model. Pentico et al. [1] relaxed one more assumption, they allowed stockouts with partial backordering in their model, and proposed EPQ with partial backordering (EPQ-PBO). Recently, a few extensions and supplements to Pentico et al.'s EPQ-PBO have been published; some of them are Pentico et al. [2], Toews et al. [3], Wee and Wang [4].

Pentico et al. [1] determine the optimal inventory policy from the three cases: to lose all sales, to meet all demand, and to meet fractional demand, in a

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way that they developed the critical value β^* of the backordering rate β , below which the optimal policy is either to meet all demand or to lose all sales, and above which the optimal policy is to allow stockouts with partial backordering and meet fractional demand. Using the same notation from Pentico et al. [1], the critical value β^* is

$$\beta^* = 1 - \sqrt{\frac{2C_0C_h'}{DC_1^2}}. \tag{1}$$

Showing that in a case when $\beta > \beta^*$ the policy of meeting fractional demand with partial backordering is not always the optimal choice, Zhang [5] amended Pentico et al.'s decision procedure [1] by developing another critical value β^{**} of the backordering rate β given by

$$\beta^{**} = \frac{PC_b(2C_0C_h' - DC_1^2)}{PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2)}. \tag{2}$$

Zhang [5] then proposed the amended decision procedure for EPQ-PBO, which we will refer to as *Zhang's procedure*, and it is as follows.

1. Determine β^* and β^{**} from (1) and (2) respectively.
2. If $\beta \leq \max(\beta^*, \beta^{**})$, compare the cost of meeting all demand (from the basic EPQ) with the cost of losing all sales. The optimal policy is the one with a lower cost.
3. If $\beta > \max(\beta^*, \beta^{**})$, the optimal policy is to meet fractional demand with partial backordering (defined by Pentico et al. [1]).

According to Zhang's procedure, when $\beta \leq \max(\beta^*, \beta^{**})$, meeting a fractional demand with partial backordering can not be an optimal decision. However, we will show that this is not always true. We will derive necessary and sufficient conditions for considering the policy of losing all sales or the policy to meet all demand as possible optimal decisions. A new decision procedure will be proposed based on these conditions. Our new procedure will be without comparison of costs.

One of the extensions of Pentico et al.'s EPQ-PBO is Pentico et al.'s EPQ-PBO and phase dependent backordering rate [2]. Relaxing the assumption on a constant all the time backordering rate β, they considered two phases of constant backordering rate. They model their decision procedure onto Pentico et al.'s decision procedure [1] by adding a comparison with cost of losing all sales when the backordering rate is above the critical value. Regarding the same model only using a different methodology, Hsieh and Dye [6] derived optimal solutions and proposed the decision procedure without comparison of costs. We will modify our new decision procedure to be applicable to this extension.

The paper is organized in the following manner. In Section 2 a numerical example is given to illustrate that decisions made with Zhang's procedure are not always optimal. A new cost comparison free decision procedure for Pentico et al.'s EPQ-PBO is proposed in Section 3. In Section 4 a new decision procedure for the extension EPQ-PBO and phase dependent backordering rate is proposed.

2 Numerical example

The example that is given in Zhang [5] illustrates that following the Zhang's procedure, one can make a right optimal decision. The values of the parameters in this example are: $D = 110$ units/year, $P = 9200$ units/year, $C_0 = \$275$ /setup, $C_h = \$2.00$ /unit/year, $C_b = \$0.70$ /unit/year, $C_1 = \$2.4$ /unit. In order to show failure of Zhang's procedure to make the right optimal decision, we will give another example. The values of the parameters in our example are: $D = 5000$ units/year, $P = 9200$ units/year, $C_0 = \$275$ /setup, $C_h = \$2.00$ /unit/year, $C_b = \$3.2$ /unit/year, $C_1 = \$0.5$ /unit. Then, according to Pentico et al. [1], the value of C'_h is

$$C'_h = C_h(1 - D/P) = 2 \cdot (1 - 5000/9200) = 0.913043.$$

The cririal values β^* and β^{**} according to (1) and (2) respectively are

$$\beta^* = 1 - \sqrt{\frac{2C_0C'_h}{DC_1^2}} = 1 - \sqrt{\frac{2 \cdot 275 \cdot 0.913043}{5000 \cdot 0.5^2}} = 0.366171,$$

$$\begin{aligned} \beta^{**} &= \frac{PC_b(2C_0C'_h - DC_1^2)}{PDC'_hC_1^2 + DC_b(2C_0C'_h - DC_1^2)} = \\ &= \frac{9200 \cdot 3.2 \cdot (2 \cdot 275 \cdot 0.913043 - 5000 \cdot 0.5^2)}{9200 \cdot 5000 \cdot 0.913043 \cdot 0.5^2 + 5000 \cdot 3.2 \cdot (2 \cdot 275 \cdot 0.913043 - 5000 \cdot 0.5^2)} = \\ &= 15.0258. \end{aligned}$$

Let $\beta = 0.5$ (note that $\beta > \beta^* = 0.366171$). According to Zhang's procedure $\beta \le \max(\beta^*, \beta^{**}) = 15.0258$, so we should compare the cost of meeting all demand

$$\Gamma^*_{EPQ} = \sqrt{2C_0C'_hD} = \sqrt{2 \cdot 275 \cdot 0.913043 \cdot 5000} = 1584.57$$

with the cost of losing all sales

$$\Gamma_{LS} = C_1D = 0.5 \cdot 5000 = 2500.$$

Since, $\Gamma^*_{EPQ} = 1584.57 < 2500 = \Gamma_{LS}$, according to Zhang's procedure, the optimal policy is to allow no stockouts.

But, if we calculate the optimal time length of the inventory cycle T^*, the optimal fill rate F^* and the value of the cost function $\Gamma(T, F)$ for $(T, F) = (T^*, F^*)$, according to the formulas given in Pentico et al. [1], we will obtain the values

$$T^* = 0.395136,\ F^* = 0.865103,\ \Gamma(T^*, F^*) = 1560.55.$$

Comparing this cost of meeting fractional demand with the cost of meeting all demand, we have

$$\Gamma(T^*, F^*) = 1560.55 < \Gamma^*_{EPQ} = 1584.57.$$

Therefore, even if $\beta \leq \max(\beta^*, \beta^{**})$, the cost of meeting fractional demand is lower than the cost of meeting all demand (which should be the lowest cost according to Zhang's procedure), so Zhang's procedure failed in making optimal decision. Just for illustration, in this case, Pentico et al.'s decision procedure [1] will make a right optimal decision.

3 A new decision procedure for EPQ-PBO

With an intention to correct Pentico et al.'s decision procedure [1], Zhang [5] derived a new critical value β^{**}, defined with (2), and the condition for meeting fractional demand when $\beta > \beta^*$, where β^* is the critical value defined with (1). By taking in consideration the cost of losing all sales as a possible optimal decision when $\beta > \beta^*$, Zhang at first transformed the inequality $\Gamma(T^*, F^*) \leq C_1 D$ into

$$\beta \geq \frac{2C_0 C_b'}{DC_1^2} - \frac{C_b'}{C_h'}, \tag{3}$$

and then into $\beta \geq \beta^{**}$, without providing any details for these transformations. But, during the second transformation Zhang [5] overlooked the sign of the expression $PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2)$, as we are going to show.

Namely, if we substitute $C_b' = C_b(1 - \beta D/P)$ into (3) we will have

$$\beta \geq \frac{2C_0 C_b(1 - \beta D/P)}{DC_1^2} - \frac{C_b(1 - \beta D/P)}{C_h'},$$

and after some algebraic transformations the last inequality is equivalent to

$$\beta(PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2)) \geq PC_b(2C_0C_h' - DC_1^2). \tag{4}$$

Now, if the expression $PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2) > 0$ then (4) implies

$$\beta \geq \frac{PC_b(2C_0C_h' - DC_1^2)}{PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2)} = \beta^{**}.$$

Otherwise, if $PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2) < 0$ then (4) implies $\beta \leq \beta^{**}$. Consequently, the derivation in Zhang [5] is correct only if $PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2) > 0$.

Another observation that is worth mentioning is that if the expression $2C_0C_h' - DC_1^2 > 0$, then the expression $PDC_h'C_1^2 + DC_b(2C_0C_h' - DC_1^2) > 0$ and Zhang's procedure will make right decisions. On the other hand, the sign of the expression $2C_0C_h' - DC_1^2$ is closely related to the sign of the critical value β^* as it is shown below,

$$2C_0C_h' - DC_1^2 > 0 \Leftrightarrow \frac{2C_0C_h'}{DC_1^2} > 1 \Leftrightarrow \beta^* = 1 - \sqrt{\frac{2C_0C_h'}{DC_1^2}} < 0. \tag{5}$$

According to (5) and the above discussion, when $\beta^* < 0$ then Zhang's procedure will make the right decisions. So, what is left is to correct Zhang's procedure when $\beta^* \geq 0$.

As it is shown below, in the case $\beta^* \geq 0$ the cost of losing all sales $\Gamma_{LS} = C_1 D$ is not lower that the cost of meeting all demand $\Gamma^*_{EPQ} = \sqrt{2C_0 C'_h D}$, and in this case to lose all sales can never be an optimal decision. We have,

$$\beta^* = 1 - \sqrt{\frac{2C_0 C'_h}{DC_1^2}} = 1 - \frac{\sqrt{2C_0 C'_h D}}{DC_1} = 1 - \frac{\Gamma^*_{EPQ}}{\Gamma_{LS}} \geq 0 \Leftrightarrow \Gamma^*_{EPQ} \leq \Gamma_{LS}. \quad (6)$$

The last equation (6) gives the necessary and sufficient condition for considering the policy of losing all sales or the policy of meeting all demand as possible optimal decisions. It states that when $\beta^* \geq 0$ decision makers should not take into consideration the policy of losing all sales as an optimal decision (consequently, there is no need of the second critical value β^{**}), and when $\beta^* < 0$ the policy of meeting all demand should not to be taken into consideration as an optimal one.

We will construct our new decision procedure upon the above reasoning given with the equation (6) and the facts about the critical values β^* and β^{**} when related to the backordering rate β from Zhang [5] when $\beta^* < 0$, and from Pentico et al. [1] when $\beta \geq \beta^*$ (note that for $\beta^* < 0$, as $0 \leq \beta \leq 1$, it is always true that $\beta \geq \beta^*$).

Now, we can propose our new cost comparison free decision procedure for Pentico et al.'s EPQ-PBO. We will refer to it as *Sign procedure.*

1. Determine β^* from (1).
2. If $\beta^* \geq 0$, compare β with β^*.
   - 2.1. If $\beta > \beta^*$, the optimal policy is to meet fractional demand with partial backordering (calculate F^*, T^* and $\Gamma(T^*, F^*)$ according to the formulas (18)-(19) in Pentico et al. [1]).
   - 2.2. If $\beta \leq \beta^*$, the optimal policy is to meet all demand (from the basic EPQ determine $T^* = \sqrt{2C_0/(DC'_h)}$ and the optimal cost $\Gamma^*_{EPQ} = \sqrt{2C_0 C'_h D}$).
3. If $\beta^* < 0$, determine β^{**} from (2) and compare β with β^{**}.
   - 3.1. If $\beta > \beta^{**}$, the optimal policy is to meet fractional demand with partial backordering (calculate F^*, T^* and $\Gamma(T^*, F^*)$ according to the formulas (18)-(19) in Pentico et al. [1]).
   - 3.2. If $\beta \leq \beta^{**}$, the optimal policy is to lose all sales (calculate the cost of losing all sales $\Gamma_{LS} = C_1 D$).

We will illustrate the Sign procedure on three examples. For the above numerical example $\beta^* = 0.366171 > 0$. Then, for $\beta = 0.5 > \beta^*$, the optimal decision according to Sign procedure is to meet fractional demand with a minimum cost of 1560.55. And, for $\beta = 0.3 < \beta^*$, the optimal decision according to Sign procedure is to meet all demand with a minimum cost of 1584.57. In this case the Sign procedure has made the same decisions as Pentico et al.'s decision procedure [1], and Zhang's procedure failed in making the right optimal decision.

For the example given in Zhang [5] $\beta^* = -0.309715 < 0$, so we should calculate $\beta^{**} = 0.252638$. Then, for $\beta = 0.3 > \beta^{**}$, the optimal decision according to Sign procedure is to meet fractional demand with minimum cost of 259.249. But, for $\beta = 0.2 < \beta^{**}$, the optimal decision according to Sign procedure is to lose all sales with minimum cost of 264. In this case the Sign procedure has made the same decisions as Zhang's procedure, and Pentico et al.'s decision procedure [1] failed in making the right optimal decision.

The example given in Pentico et al. [1] has the following values for the parameters: $D = 1100$ units/year, $P = 9200$ units/year, $C_0 = \$275$ /setup, $C_h = \$2.00$ /unit/year, $C_b = \$3.2$ /unit/year, $C_1 = \$4$ /unit. For this example $\beta^* = 0.765421 > 0$ and $\beta^{**} = -2.161 < \beta^*$. So, all three procedures Pentico et al.'s decision procedure [1], Zhang's procedure and Sign procedure will make same decisions (to look at the optimal decisions in this case for different values of β, see [1]).

If the production rate is infinitely large, the Pentico et al.'s EPQ-PBO [1] will degenerate into Pentico and Drake's EOQ-PBO [7]. When we substitute C_h for C'_h, C_b for C'_b and for β^{**} substitute $\lim_{P\to\infty}\beta^{**} = (C_b(2C_0C_h - DC_1^2))/(DC_hC_1^2)$, then the Sign procedure can be used for Pentico and Drake's EOQ-PBO [7].

The decision procedure formed upon critical values of the backordering rate is more practical to implement because it needs less computational effort and when critical values are calculated once for some given values for model parameters, it is easy to check for the optimal policy for different values of the backordering rate. However, sometimes it is difficult to calculate the critical value, for instance in case of LIFO service rule, the critical value β^* is very difficult to derive (Pentico et al. [1]), and the critical value β^{**} is almost impossible to obtain (Zhang [5]). In this case, it is more convenient in the decision procedure to impose inequalities with respect to the backordering rate β, as Zhang [5] suggested in case of LIFO.

Taking into consideration the above discussion, we can give a more concise correction for Pentico et al.'s decision procedure [1] without deriving the second critical value. As we mentioned before, the first part of the transformation that was done in Zhang [5], in order to obtain the critical value β^{**}, is correct i.e. $\Gamma(T^*, F^*) \leq C_1D$ is equivalent to $\beta \geq 2C'_bC_0/(DC_1^2) - C'_b/C'_h$ which is previously noted as inequality (3). We will use the inequality (3) to decide between meeting the fractional demand and losing all sales as an optimal policy when $\beta > \beta^*$. And when $\beta \leq \beta^*$ we do not need to compare the cost of meeting all demand with the cost of losing all sales as Pentico et al.'s decision procedure [1] requires. Knowing that $0 \leq \beta \leq 1$ and $\beta \leq \beta^*$ implies that $\beta^* \geq 0$, and according to the previously derived equation (6) we have that the cost of meeting all demand is less than the cost of losing all sales. So, when $\beta \leq \beta^*$ the only optimal decision is to meet all demand. Hence, the more concise decision procedure for Pentico et al.'s EPQ-PBO will be as follows. We will refer to it as *corrected P procedure.*

1. Determine β^* from (1).
2. If $\beta \leq \beta^*$, the optimal policy is to meet all demand.
3. If $\beta > \beta^*$ then, if the inequality (3) holds, the optimal policy is to meet fractional demand with partial backordering. Otherwise, the optimal policy is to lose all sales.

We can simplify the computation in Step 3 of the corrected P procedure, if we check the equivalent inequality $\beta \geq (C_b'/C_h')\beta^*(\beta^* - 2)$ instead of checking the inequality (3), since the value of β^* has already been computed. Note that, if $\beta^* > 0$, then this condition is clearly satisfied, since the term of the right is negative.

4 A new decision procedure for an extension of EPQ-PBO

One of the extensions of Pentico et al.'s EPQ-PBO is Pentico et al.'s EPQ-PBO and phase dependent backordering rate [2]. Relaxing the assumption on a constant all the time backordering rate β, they considered two phases of constant backordering rate. During the first phase, before the start of production, the backordering rate is β, and during the second phase, after production starts, the backordering rate is $\rho\beta$, where $1 \leq \rho \leq 1/\beta$. Note that for $\rho = 1$ this extension coincides with Pentico et al.'s EPQ-PBO [1]. We will modify the Sign procedure to be applicable to the extension.

Using the same notation from Pentico et al. [2], the critical value of the backordering rate β developed by Pentico et al. [2], which we will denote with β^*_{new}, below which the optimal policy is either to meet all demand or to lose all sales, and above which the optimal policy is either to allow stockouts with partial backordering and meet fractional demand or to lose all sales is

$$\beta^*_{new} = \frac{\beta^*}{1 + (\rho - 1)(D/P)\beta^*}, \tag{7}$$

where β^* is defined with (1). Then, the optimality of policies are determined by comparison of costs.

In order to compose a cost comparison free decision procedure and still be able to maintain the critical values of the backordering rate, we will modify the Sign procedure. The equation (6) which provides necessary and sufficient conditions for considering the policy of meeting all demand or the policy of losing all sales as possible optimal policies, will still be the essential ingredient in the new modified procedure. When $\beta^* \geq 0$ we will decide between meeting fractional demand and meeting all demand by comparing the backordering rate β with Pentico et al.'s [2] critical value β^*_{new} defined with (7). But, when $\beta^* < 0$ we need to develop another critical value, analogous to β^{**} defined with (2).

When $\beta^* < 0$, we need to choose between meeting fractional demand and losing all sales, depending on the amount of cost. Similarly as in Zhang [5], a new critical value of the backordering rate can be derived from the inequality $\Gamma(T^*, F^*) < C_1 D$, where $\Gamma(T^*, F^*)$ is the value of the cost function when fractional demand is met calculated at the solution (T^*, F^*) from Pentico et al. [2]. The last inequality is equivalent to

$$2C_0 C'_h - DC_1^2 - \frac{DC_1^2 C'_h \beta}{C'_b(1 - (\rho - 1)\beta D/P)^2} < 0. \tag{8}$$

Two cases should be distinguished, $\rho = 1$ and $\rho > 1$. When $\rho = 1$, the inequality (8) is equivalent to

$$\beta > \frac{PC_b(2C_0 C'_h - DC_1^2)}{PDC'_h C_1^2 + DC_b(2C_0 C'_h - DC_1^2)} = \beta^{**}, \tag{9}$$

which is as expected since for $\rho = 1$ the observed model is same as Pentico et al.'s EPQ-PBO [1]. When $\rho > 1$, inequality (8) is equivalent to

$$\beta > \beta^{**}_{new}(\rho) = \frac{\frac{D}{P} + A + 2(\rho - 1)\frac{D}{P} - \sqrt{\left(\frac{D}{P} + A\right)^2 + 4(\rho - 1)\frac{D}{P}A}}{2\rho(\rho - 1)\frac{D^2}{P^2}}, \tag{10}$$

where

$$A = \frac{DC_1^2 C'_h}{C_b(2C_0 C'_h - DC_1^2)}. \tag{11}$$

Using the notation (11), we can define the critical value $\beta^{**}_{new}(\rho)$ for $\rho = 1$ with

$$\beta^{**}_{new}(1) = \beta^{**} = \frac{1}{\frac{D}{P} + A}. \tag{12}$$

We can see that the critical value $\beta^{**}_{new}(\rho)$ is well defined for all $1 \le \rho \le 1/\beta$ since $\lim_{\rho \to 1} \beta^{**}_{new}(\rho) = 1/(D/P + A) = \beta^{**} = \beta^{**}_{new}(1)$.

Now, we can state our new modified cost comparison free decision procedure for Pentico et al.'s EPQ-PBO and phase dependent backordering rate as follows. We will refer to it as *modified Sign procedure.*

1. Determine β^* from (1).
2. If $\beta^* \ge 0$, determine β^*_{new} from (7) and compare β with β^*_{new}.
   - 2.1. If $\beta > \beta^*_{new}$, the optimal policy is to meet fractional demand with partial backordering (calculate F^*, T^* and $\Gamma(T^*, F^*)$ according to the formulas (9)-(11) in Pentico et al. [2]).
   - 2.2. If $\beta \le \beta^*_{new}$, the optimal policy is to meet all demand (from the basic EPQ determine $T^* = \sqrt{2C_0/(DC'_h)}$ and the optimal cost $\Gamma^*_{EPQ} = \sqrt{2C_0 C'_h D}$).

3. If $\beta^* < 0$, determine $\beta^{**}_{new}(\rho)$ from (10)-(12) and compare β with $\beta^{**}_{new}(\rho)$.

   3.1. If $\beta > \beta^{**}_{new}(\rho)$, the optimal policy is to meet fractional demand with partial backordering (calculate F^*, T^* and $\Gamma(T^*, F^*)$ according to the formulas (9)-(11) in Pentico et al. [2]).

   3.2. If $\beta \leq \beta^{**}_{new}(\rho)$, the optimal policy is to lose all sales (calculate the cost of losing all sales $\Gamma_{LS} = C_1 D$).

We are going to illustrate the modified Sign procedure on the example from Zhang [5] for $\beta = 0.2$ and $\rho = 1.5$, which means that the percentage of demand backordered increases from 0.2 to 0.3 after production begins. At first, the critical value β^* calculated from (1) is $\beta^* = -0.309715 < 0$, and according to the procedure we should calculate $\beta^{**}_{new}(\rho)$ from (10)-(11) for $\rho = 1.5 > 1$. We have $A = 3.94628$, and $\beta^{**}_{new}(\rho) = 0.251879$. Since $\beta = 0.2 < \beta^{**}_{new}(\rho) = 0.251879$ according to the procedure the optimal policy is to lose all sales with optimal cost $\Gamma_{LS} = 264$.

As mentioned previously, if $\rho = 1$ this model will degenerate into Pentico et al.'s EPQ-PBO with constant all the time backordering rate [1], and the modified Sign procedure will be same as the Sign procedure. If $\rho = 1$ and the production rate is infinitely large, this model will degenerate into Pentico and Drake's EOQ-PBO [7], and when we substitute C_h for C'_h, C_b for C'_b, $C_1(1-\beta)$ for C'_1, then for β^*_{new} substitute $\lim_{P\to\infty} \beta^*_{new} = 1 - \sqrt{(2C_0C_h)/(DC_1^2)}$ and for $\beta^{**}_{new}(\rho)$ substitute $\lim_{P\to\infty} \beta^{**} = (C_b(2C_0C_h - DC_1^2))/(DC_hC_1^2)$, then the modified Sign procedure can be used for Pentico and Drake's EOQ-PBO [7].

In order to provide a more concise decision procedure as we did in the previous section, we will use the inequality (8) to decide between meeting the fractional demand and losing all sales when $\beta > \beta^*_{new}$. But, when $\beta \leq \beta^*_{new}$ we will check the sign of β^* to decide between the policy of meeting all demand and the policy of losing all sales as an optimal one, according to the derived equation (6). Hence, another decision procedure for Pentico et al.'s EPQ-PBO and phase dependent backordering rate that we will refer to as *modified P procedure* is as follows.

1. Determine β^* from (1) and β^*_{new} from (7).

2. If $\beta \leq \beta^*_{new}$ then, if $\beta^* \geq 0$ the optimal policy is to meet all demand. Otherwise, the optimal policy is to lose all sales.

3. If $\beta > \beta^*_{new}$ then, if the inequality (8) holds, the optimal policy is to meet fractional demand with partial backordering. Otherwise, the optimal policy is to lose all sales.

It is apparent that the above modified P procedure needs more computational efforts, when the checking of the inequality (8) is an issue. The computation in Step 3 of the modified P procedure can be simplified, as it was done within corrected P procedure. Instead of checking the inequality (8), we can check the equivalent inequality $\beta/(1-\beta(1/\beta^*_{new} - 1/\beta^*))^2 > (C'_b/C'_h)\beta^*(\beta^* - 2)$, since the values of β^* and β^*_{new} have already been computed. And if $\beta^* > 0$, then this condition is clearly satisfied, since the term of the right is negative and the term of the left is always positive.

5 Conclusions

In Zhang [5], the author proposed the amended decision procedure for the Pentico et al.'s EPQ-PBO [1], by developing another critical value of the backordering rate. In this paper we showed that this amended decision procedure does not always make optimal decisions. We found out what caused such irregularity and fixed it by deriving necessary and sufficient conditions for considering the policy of losing all sales or the policy of meeting all demand as possible optimal decisions. We proposed a new cost comparison free decision procedure based on these conditions. Then we modified this procedure in order for it to be applicable to the Pentico et al.'s EPQ-PBO and phase dependent backordering rate [2]. These procedures were compared to more concise ones that avoid derivation of second critical values for the backordering rate and check the inequalities with respect to the backordering rate.

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